

Observational tests of inflation*

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Abstract

We are on the verge of the first precision testing of the inflationary cosmology as a model for the origin of structure in the Universe. I review the key predictions of inflation which can be used as observational tests, in the sense of allowing inflation to be falsified. The most important prediction of this type is that the perturbations will cross inside the Hubble radius entirely in their growing mode, though nongaussianity can also provide critical tests. Spatial flatness and tensor perturbations may offer strong support to inflation, but cannot be used to exclude it. Finally, I discuss the extent to which observations will distinguish between inflation models, should the paradigm survive these key tests, in particular describing a technique for reconstruction of the inflaton potential which does not require the slow-roll approximation.

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1 What does inflation predict?

Cosmological inflation [1, 2, 3] is widely perceived as an excellent paradigm within which one can explain both the global properties of the Universe and the irregularities which give rise to structures within it. Despite this, it remains fair to say that as yet the inflationary paradigm has been confronted with only a few observational challenges, which it has comfortably surmounted. In years to come, it will face many more, and the purpose of this article is to discuss which of these tests are likely to be the most stringent.

In doing so, it is worthwhile to separate out the two key roles that inflation plays in modern cosmology. The first, which led to its introduction, is in setting the ‘initial conditions’ for the global Universe, by arranging a large homogeneous Universe devoid of unwanted relics such as monopoles. In terms of these global properties, it now seems unlikely that any new observations will undermine the inflationary picture, and, as Linde has argued [4], if it is to be supplanted that is likely to be because of the advent of a superior theory, rather than of superior observations. Accordingly, I will have little to say on this topic.

The second role, which is potentially much more fruitful as a probe of high-energy physics, is that inflation provides a theory for the origin of perturbations in the Universe (for reviews, see Refs. [2, 5]). As these perturbations are believed to evolve into all the observed structures in the present Universe, including the existence and clustering of galaxies and the anisotropies in the cosmic background radiation, this proposal is subject to a wide variety of observational tests. Thus far, these tests have been rather qualitative in nature, but in the near future inflation as a theory of the origin of structure in the Universe will face precision testing.

The challenge facing cosmologists is therefore to address two questions:

- Is inflation right?
- If so, which version of inflation is right?

Unfortunately, in science one never gets to prove that a theory is correct, merely that it is the best available explanation. The way to convince the community that a theory is indeed the best explanation is if that theory can repeatedly pass new observational tests. In that regard, it is important to be as clear as possible concerning what these tests might be.

2 The predictions of inflation

The essence of testing inflation can be condensed into a single sentence, namely

The *simplest* models of inflation predict power-law spectra of gaussian adiabatic scalar and tensor perturbations in their growing mode in a spatially-flat Universe.

This sentence contains 6 key predictions of the inflationary paradigm, which I've underlined, but also one crucial word, 'simplest'. The trouble is that inflation is a paradigm rather than a model, and has many different realizations which can lead to a range of different predictions. From a straw poll of cosmologists, everyone agreed that there were at least several tens of different models, and I'd say there certainly aren't as many as a thousand, so a reasonable first guess is that at present there are around one hundred different models on the market, all consistent (at least more or less) with present observational data.

A valuable scientific theory is one which has sufficient predictive power that it can be subjected to observational tests which are capable of falsifying it. When the model survives such a test, it strengthens our view that the model is correct; in Bayesian terms, its likelihood is increased relative to models which are less capable of matching the data. It is useful to think of these models at three different levels:

- **Specific models of inflation.** These are readily testable. For example, there will be a specific prediction for the spectral index n of the density perturbations, and this will be measured to high accuracy.
- **Classes of models.** This means models sharing some common property, for example that the perturbations are gaussian. An entire class of models can be excluded by evidence against that shared property.
- **The inflationary paradigm.** Testing the paradigm as a whole requires a property which is robust amongst all models. As we'll see, the most striking such property is that the perturbations should be in their growing mode, which leads to the distinctive signature of oscillations in the microwave anisotropy power spectra.

Finally, note that since one can never completely rule out a small inflationary component added on to some rival structure formation model (e.g. a combined cosmic strings and inflation model [6]), in practice we are initially testing the paradigm of inflation as the *sole* origin of structure in the Universe.

It can also be helpful to make the admittedly rather narrow distinction between tests and supporting evidence [7]. A test arises when there is a prediction which, if contradicted by observations, rules out the model, or at least greatly reduces its likelihood relative to a rival model. In this sense, the geometry of the Universe is not a test of inflation, because there exist inflation models predicting whatever geometry might be measured (including open and closed ones), and there is no rival regarded as giving a better explanation for any particular possible observation. By contrast, the oscillations in the microwave anisotropy power spectra (both temperature and polarization) do give rise to a test, as we will shortly see.

Supporting evidence arises with observational confirmation of a prediction which is regarded as characteristic, but which is not generic. A good example would be the observation of tensor perturbations with wavelengths exceeding the Hubble length, for which inflation would be by far the best available explanation; they do not give rise to a test because if they are not observed, then there are plentiful inflation models

where such perturbations are predicted to be below any anticipated observational sensitivity.

3 Testing the predictions

3.1 Spatial flatness

Of the listed properties, spatial flatness is the only one which refers to the global properties of the Universe.¹ It is particularly pertinent because of the original strong statements that spatial flatness was an inevitable prediction of inflation, later retracted with the discovery of a class of models — the open inflation models [8, 9] — which cunningly utilize quantum tunnelling to generate homogeneous open Universes. In the recent ‘tunnelling from nothing’ instanton models of Hawking and Turok [9], any observed curvature has the interesting interpretation of being a relic from the initial formation of the Universe which managed to survive the inflationary epoch.

If we convince ourselves that, to a high degree of accuracy, the Universe is spatially flat, that will strengthen the likelihood that the simplest models of inflation are correct. However, an accurate measurement of the curvature is not a *test* of the full inflationary paradigm, because whatever the outcome of such a measurement there do remain inflation models which make that prediction. This point has recently been stressed by Peebles [7]. The likelihood will have shifted to favour some inflation models at the expense of others, but the total likelihood of inflation will be unchanged.² Only if a rival class of theories can be invented, which predict say a negative-curvature Universe in a way regarded as more compelling than the open inflation models, will measurements of the curvature acquire the power to test the inflationary paradigm.

I should also mention that the standard definition of inflation — a period where the scale factor $a(t)$ undergoes accelerated expansion — is a rather general one, and in particular any classical solution to the flatness problem using general relativity must involve inflation. This follows directly from writing the Friedmann equation as

$$|\Omega - 1| = \frac{|k|}{\dot{a}^2}. \quad (1)$$

An example is the pre big bang cosmology [11], which is now viewed as a novel type of inflation model rather than a separate idea. This makes it hard to devise alternative solutions to the flatness problem; open inflation models use quantum tunnelling but in fact still require classical inflation after the tunnelling, and presently the only existing alternative is the variable-speed-of-light theories [10] which violate general relativity.

¹Inflation is also responsible for solving the horizon problem, ensuring a Universe close to homogeneity, but this is no longer a useful test as it is already observationally verified to high accuracy through the near isotropy of the cosmic microwave background.

²Indeed, the only existing alternative to inflation in explaining spatial flatness is the variable-speed-of-light (VSL) theories [10], which may be able to solve the problem without inflation, though at the cost of abandoning Lorentz invariance. There are no available alternatives at all to inflation in explaining an open Universe, so one might say that observation of negative curvature modestly *improves* the likelihood of inflation amongst known theories, by eliminating the VSL theories from consideration.

Before continuing on to the properties of perturbations in the Universe, there's a final point worth bearing in mind concerning inflation as a theory of the global Universe. As I've said, there now seems little prospect that any observations will come along which might rule out the model. But it is interesting that while that is true now, it was *not* true when inflation was first devised. An example is the question of the topology of the Universe. We now know that if there is any non-trivial topology to the Universe, the identification scale is at least of order the Hubble radius, and I expect that that can be consistent with inflation (though I am unaware of any detailed investigation of the issue). However, from observations available in 1981 it was perfectly possible that the identification scale could have been much much smaller. Since inflation will stretch the topological identification scale, that would have set an upper limit on the amount of inflation strong enough to prevent it from solving the horizon and flatness problems. The prediction of no small-scale topological identification has proven a successful one. Another example of a test that could have excluded inflation, but didn't, is the now-observed absence of a global rotation of our observable Universe [12].

3.2 Growing-mode perturbations

This is the key prediction of inflation as a theory of the origin of structure; inflation generically predicts oscillations in the temperature and polarization angular power spectra. If oscillations are not seen, then inflation cannot be the sole origin of structure.

The reason this prediction is so generic is because inflation creates the perturbations during the early history of the Universe, and they then evolve passively until they enter the horizon in the recent past. The perturbations obey second-order differential equations which possess growing and decaying mode solutions, and by the time the perturbations enter the horizon the growing mode has become completely dominant. That means the solution depends only on one parameter, the amplitude of the growing mode; in particular, the derivative of the perturbation is a known function of the amplitude. The solution inside the horizon is oscillatory before decoupling, and this fixes the temporal phase of the perturbations; all perturbations of a given wavenumber oscillate together and in particular at any given time there are scales on which the perturbation vanishes. Projected onto the microwave sky, this leads to the familiar peak structure seen in predicted anisotropy spectra, though if one wants to be pedantic the troughs are if anything more significant than the peaks.

The importance of the peak structure in distinguishing inflation from rivals such as defect theories was stressed by Albrecht and collaborators [13] and by Hu & White [14]. The prediction is a powerful one; in particular it still holds if the inflationary perturbations are partly or completely isocurvature, and if they are nongaussian.

I stress that while inflation inevitably leads to the oscillations, I am not saying that inflation is the *only* way to obtain oscillations. For example there are known active source models which give an oscillatory structure [15], though the favourite cosmic string model is believed not to. If observed, oscillations would support inflation but cannot prove it. However I might mention in passing that it is quite easy to prove [16]

that the existence of adiabatic perturbations on scales much larger than the Hubble radius would imply one of three possibilities; the perturbations were there as initial conditions, causality/Lorentz invariance is violated, or a period of inflation occurred in the past.

Against designer inflation

At this point it is worth saying something against ‘designer’ models of inflation which aim to match observations through the insertion of features in the spectra, by putting features in the inflationary potential. This idea first arose in considering the matter power spectrum [17], which is a featureless curve and so quite amenable to the insertion of peaks and troughs. However the idea is much more problematic in the context of the microwave anisotropy spectra, which themselves contain sharp oscillatory features. One might contemplate inserting oscillations into the initial power spectrum in such a way as to ‘cancel out’ the oscillatory structure, but there are however three levels of argument against this:

1. It’s a silly idea, because the physics during inflation has no idea where the peaks might appear at decoupling, and for the idea to be useful they have to match to very high accuracy. That argument is good enough for me, though perhaps not for everyone, so ...
2. Even if you wanted to do it you probably cannot. Barrow and myself [12] found that the required oscillations were so sharp as to be inconsistent with inflation taking place, at least in single-field inflation models. However, a watertight case remains to be made on this point, and it is not clear how one could extend that claim to multi-field models, so perhaps the most pertinent argument is ...
3. Even if you managed to cancel out the oscillations in the temperature power spectrum, the polarization spectra have oscillations which are out of phase with the temperature spectrum, and so those oscillations will be enhanced [18].

3.3 Gaussianity and adiabaticity

While the simplest models of inflation predict gaussian adiabatic perturbations, many models are known which violate either or both of these conditions. Consequently there is no critical test of inflation which can be simply stated. Nevertheless, it is clear that these could lead to tests of the inflationary paradigm. For example, as far as inflation is concerned, there is good nongaussianity and bad nongaussianity. For example, if line discontinuities are seen in the microwave background, it would be futile to try and explain them using inflation rather than cosmic strings. On the other hand, nongaussianity with a chi-squared distribution is very easy to generate in inflation models; one only has to arrange that the leading contribution to the density comes from the square of a scalar field perturbation. Indeed, in isocurvature inflation models, it appears at least as easy to arrange chi-squared statistics as it is to arrange gaussian ones [19].

Inflation may also be able to explain nongaussian perturbations of a ‘bubbly’ nature, by attributing the bubbles to a phase transition bringing inflation to an end. The simplest models of this type have already been excluded, but more complicated ones may still be viable.

3.4 Tensor and vector perturbations

Gravitational wave perturbations, also known as tensor perturbations, are inevitably produced at some level by inflation, but the level depends on the model under consideration and it is perfectly possible, and perhaps even likely [20], that the level is too small to be detected by currently envisaged experiments. This prevents them acting as a test.

In standard inflation models, the gravitational waves are directly observable only by the microwave background anisotropies they induce. Assuming Einstein gravity, the Hubble parameter always decreases during inflation which leads to a spectrum which decreases with decreasing scale; the upper limit set by these anisotropies places the amplitude on short scales orders of magnitude below planned detectors (and probably well below the stochastic background from astrophysical sources). The exception is the pre big bang class of models [11] (implemented in extensions of Einstein gravity), where the gravitational wave spectrum rises sharply to short scales and is potentially visible in laser interferometer experiments.

As well as the direct effect on the microwave background, gravitational waves evidence themselves as a deficit of short-scale power in the density perturbation spectrum of COBE-normalized models. Presently the combination of large-scale structure data with COBE gives the strongest upper limit on the fractional contribution r on COBE scales, at $r \lesssim 0.5$ [21]. There is no evidence to favour the tensors, but this constraint is fairly weak. Eventually the PLANCK satellite is expected to be able to detect (at 95% confidence) a contribution above $r \sim 0.1$ [22], and may perhaps do better if there is early reionization and/or the foreground contamination turns out to be readily modellable. Conceivably, high-precision observations of the polarization of the microwave background might improve this further.

The verdict, therefore, is that if a tensor component is seen, corresponding to gravitational waves on scales bigger than the Hubble radius at decoupling, that is extremely powerful support for the inflationary paradigm. This would be stronger yet if the observed spectrum could be shown to satisfy an equation known as the consistency equation to some reasonable accuracy; this relates the tensor spectral index to the relative amplitude of tensors and scalars, and signifies the common origin of the two spectra from a single inflationary potential $V(\phi)$ [23, 24]. However, the tensor perturbations do not provide a test for inflation in the formal sense, since no damage is inflicted upon the inflationary paradigm if they are not detected.

While known inflationary models generate both scalar and tensor modes, it appears extremely hard to generate large-scale vector modes. There are two obstacles. The first is that massless vector fields are conformally invariant, which means that perturbations are not excited by expansion; this has to be evaded either by introducing a mass (which suppresses the effect of perturbations) or an explicit coupling

breaking conformal invariance [25]. The second obstacle is that vector perturbations die off rapidly as the Universe expands, and to survive until horizon entry their initial value would have to be considerably in excess of the linear regime. In consequence, a significant prediction of inflation is the absence of large-scale vector perturbations. If they are seen, it seems likely to be impossible to make them with inflation alone, though I am not aware of a cast-iron proof. By contrast, topological defect models generically excite vector perturbations.

4 Discriminating inflationary models

4.1 Power-law behaviour

If the inflationary paradigm survives the tests above, it will be time to decide which of the existing inflation models actually fits the data. In most models, to a good approximation the density perturbations are given by a power-law and the gravitational waves are at best marginally detectable by PLANCK. Accurate measures of these two quantities have the potential to exclude nearly all existing inflation models.

At present, the spectral index is very loosely constrained; in general the limits are probably around $0.8 < n < 1.3$, though if specific assumptions are made (e.g. critical matter density or significant gravitational waves) this can tighten. As it happens, this entire viable range is fairly well populated by inflation models, which means that any increase in observational sensitivity has the power to exclude a significant fraction of them.

A benchmark for future accuracy is the PLANCK satellite; recently a detailed analysis, including estimates of foreground removal efficiency, concluded that it would reach a 1-sigma accuracy on n of around ± 0.01 [22]. By contrast, the 1-sigma error from the MAP satellite is predicted to be in the range 0.05 to 0.1, which in itself may not significantly impact on inflationary models, though it may be powerful in combination with probes such as the power spectrum from the Sloan Digital Sky Survey.

4.2 Deviations from the power-law

The power-law approximation to the spectra, as derived in Ref. [23], is particularly good at the moment because the available observations are not very accurate. In most models the spectra are indistinguishable from power laws even at PLANCK accuracy, but there are exceptions, and if deviations are observable they correspond to extra available information on the inflationary spectrum [26, 27]. One such class of models are models where features have been deliberately inserted into the potential in order to generate sharp features in the power spectrum, such as the broken scale-invariance models.

However, even without a specific feature, it may be possible to see deviations, if the slow-roll approximation is not particularly good. There is actually modest theoretical prejudice in favour of this, because in supergravity models the inflaton is

expected to receive corrections to its mass which are large enough to threaten slow-roll [3]. Specifically, the slow-roll parameter η , which is supposed to be small, receives a contribution of

$$\eta = 1 + \text{‘something’}, \quad (2)$$

where the ‘something’ is model dependent. It is clear that if slow-roll is to be very good, $\eta \ll 1$, then the ‘something’ has to cancel the ‘1’ to quite high accuracy, and there is no theoretical motivation saying it should.

If we accept that, then we conclude that n should not be extremely close to one (which would exacerbate the need for cancellation), and also that the deviation from scale-invariance, $dn/d\ln k$, which is given by the slow-roll parameters, might be large enough to be measurable [27]. A specific example where this is indeed the case is the running mass model [28, 29].

4.3 Reconstruction without slow-roll

Eventually, in order to get the best possible constraints on inflation one will want to circumvent the slow-roll approximation completely, and this can be done by computing the power spectra (first the scalar and tensor spectra, and from them the induced microwave anisotropies) entirely numerically. Such an approach was recently described by Grivell and Liddle [30], and represents the optimal way to obtain constraints on inflation from the data (though at present it has only been implemented for single-field models).

In this approach, rather than estimating quantities such as the spectral index n from the observations, one directly estimates the potential, in some parametrization such as the coefficients of a Taylor series. An example is shown in Figure 1, which shows a test case of a $\lambda\phi^4$ potential as it might be reconstructed by the PLANCK satellite — see Ref. [30] for full details. Twenty different reconstructions are shown (corresponding to different realizations of the random observational errors), whereas in the real world we would get only one of these. We see considerable variation, which arises because the overall amplitude can only be fixed by detection of the tensor component, which is quite marginal in this model. However, there are functions of the potential which are quite well determined. The lower panel shows $V'/V^{3/2}$ (where the prime is a derivative with respect to the field), which is given to an accuracy of a few percent on the scales where the observations are most efficient. This particular combination is favoured because it is the combination which (at least in the slow-roll approximation) gives the density perturbation spectrum.

No doubt, when first confronted with quality data people will aim to determine n , r , and so on along with the cosmological parameters such as the density and Hubble parameter. However, if we become convinced that the inflationary explanation is a good one, this direct reconstruction approach takes maximum advantage of the data in constraining the inflaton potential.

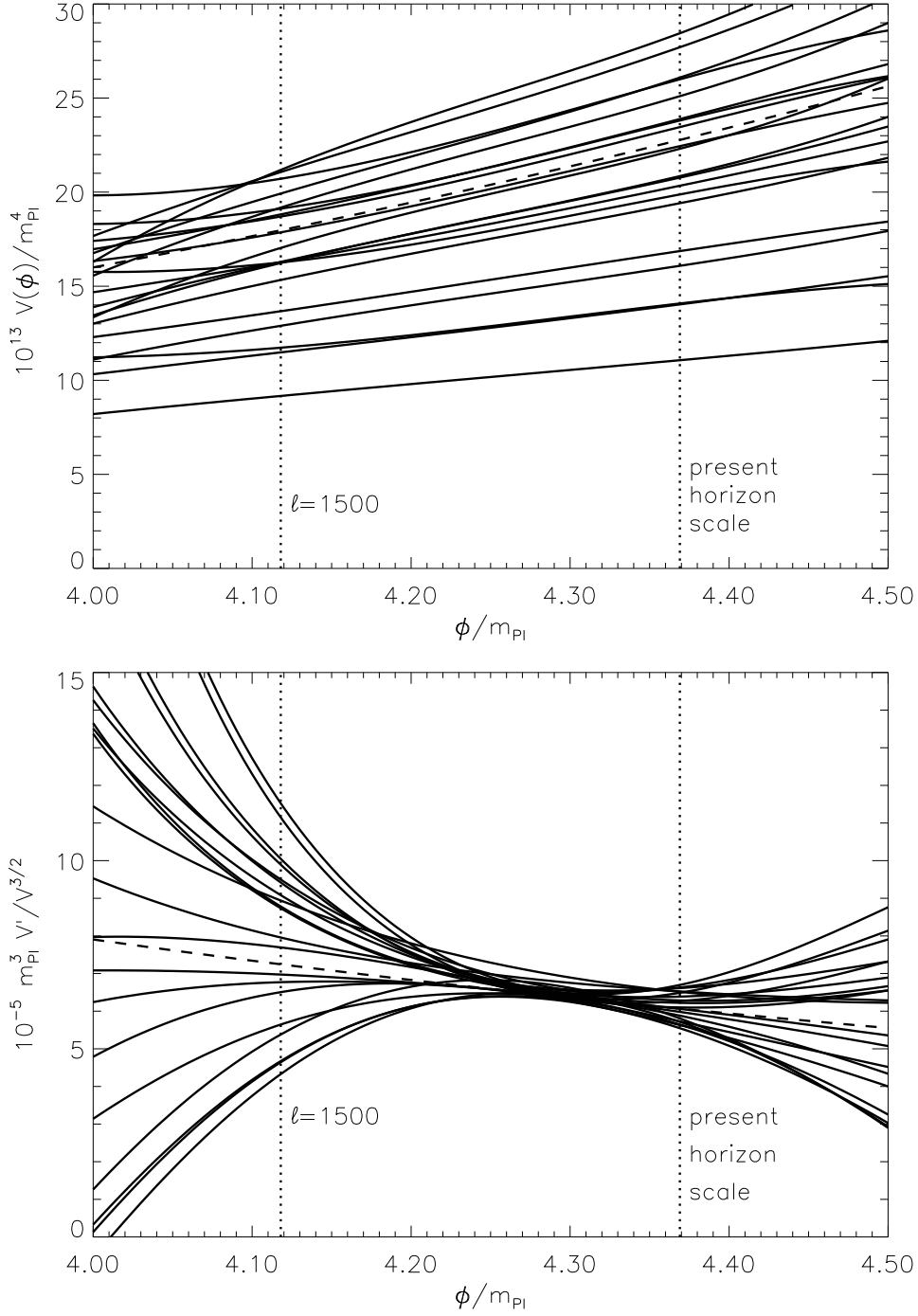


Figure 1: The $\lambda\phi^4$ potential as seen by the PLANCK satellite. In the upper panel, the dashed line shows the true potential, and the full lines show a series of Monte Carlo reconstructions, which differ in the realization of the observational errors. In reality we get only one of these curves. The lower panel shows the combination $V'/V^{3/2}$, which is much better determined. Scalar field values when scales equalled the Hubble radius during inflation are shown, roughly corresponding to the range of PLANCK.

5 Outlook

While the present situation is extremely rosy for inflation, which stands as the favoured model for the origin of structure, there is a sense in which the present is the worst time to be considering inflation models. A quick survey of the literature suggests that there are perhaps of order 100 viable models of inflation, the most there has ever been. At the original *Inner Space, Outer Space* meeting in 1984, there were only a handful. It's true that some models devised in those 15 years have been excluded, such as the extended inflation models [31, 23], but model builders have for the most part had quite a free hand operating within the given constraints.

Further, this is likely to be about the most viable models there will ever be, because observations are at the threshold of significantly impacting on this collection. Experiments such as Boomerang, VSA and MAP are capable of ruling out inflation completely, by one of the methods outlined in this article. If inflation survives, they will have significantly reduced the number of models, and then a few years later PLANCK should eliminate most of the rest. Hopefully, by the time of *Inner Space, Outer Space III* in 2014, we will be back once more to a handful.

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